Roll No. .... Total Printed Pages -11

F - 1445

C.B.S. (Sixth Semester)
EXAMINATION, May - June, 2022
ANALYSIS-IV
(For Mathematics Stream)
(M-601)

Time : Three Hours] *[Maximum Marks:40*

Note: Attempt all sections as directed.

(Section-A)

(Objective/Multiple Choice Questions)
(0.5 mark each)

Note : Attempt all questions.

Choose the correct option :

1. If |a|<1 &|b|<1, then $\left|\frac{a-b}{1-\bar{a}b}\right| is$

   (A) equal to 1

   (B) less than 1

   (C) greater than 1

   (D) equal to zero

2. If Z_1 and Z_2 are two complex numbers, then which of the following is true?

   (A) $|Z_1 - Z_2| \leq ||Z_1| - |Z_2||$

   (B) $|Z_1 - Z_2| \geq ||Z_1| - |Z_2||$

   (C) $|Z_1 - Z_2| = ||Z_1| - |Z_2||$

   (D) $|Z_1 - Z_2| \leq |Z_1| - |Z_2|$

3. Fixed points of bilinear transformation $\omega = \frac{z}{2-z}$ are

   (A) 0,1

   (B) 2,3

   (C) 1,-1

   (D) 0,2

4. Every bilinear transformation can be expressed as a resultant of

   (A) an odd number of inversions

   (B) an even number of inversions

   (C) both (A) and (B)

   (D) None of these

5. If harmonic functious u and v satisfy Cauchy- Reimann equation, then u+iv is

(A) an entire function

(B) a constant function

(C) an analytic function

(D) None of these

6. The radius of convergence of power series $\Sigma \frac{(n!)^2}{(2n)!} z^n$

(A) 1

(B) 2

(C) 3

(D) 4

7. Every analytic function can be developed in a

(A) Convergent Taylor's series

(B) Divergent series

(C) infinite series

(D) None of these

8. Zeros of an analytic function are

(A) Critical points

(B) Fixed points

(C) Isolated points

(D) Singular points

9. $\int_L |dz|$ where L is any rectifiable arc joining the points z=a and z=b is equal to

(A) |b-a|

(B) (b-a)

(C) arc length of L

(D) 0

10. If the principal part of Laurent's series is zero, then the Laurent's series reduces to

(A) Maclaurin's series

(B) Cauchy's series

(C) Taylor's series

(D) None of these

11. Value of $\int_C \frac{z^2 - z + 1}{z - 1}$, where C is the circle |z|=1

(A) 0

(B) πi

(C) $-2\pi i$

(D) $2\pi i$

12. If a function is analytic at all the points of a bounded domain except of finitely many points, then these points are called

    (A) Singular points

    (B) Simple points

    (C) Continuous points

    (D) None of these

13. Every polynomial of degree n has exactly

    (A) (n-1) zeros

    (B) n-zeros

    (C) (n+1) zeros

    (D) exactly one zero

14. The zero of first order is known as

    (A) complex zero

    (B) simple zero

    (C) singularity

    (D) None of these

15. z=1 is _____ pole of $f(z)=\frac{1}{z(z-1)^2}$

    (A) Zero

    (B) Simple pole

    (C) Double pole

    (D) None of these

16. No. of roots of the equation $z^8-4z^5+z^2-1=0$, that lie inside the circle $|z|=1$ is.....

    (A) 8

    (B) 5

    (C) 2

    (D) 1

17. For the function f (z)= $e^{1/2}$, the point z=0 is its

    (A) Removable singularity

    (B) Isolated singularity

    (C) Essential singularity

    (D) Pole

18. A rational function has no singularities other than

    (A) Removable singularity

    (B) Isolated singularities

    (C) Essential singularities

    (D) Poles

19. If $f:G\rightarrow\mathbb{C}$ is an analytic function, then u-Ref, and v= Imf are called

    (A) Laplace equation

    (B) Harmonic conjugates

    (C) Harmonic function

    (D) None of these

20. A continuous function $u : G \to R,$ which has the mean value property, then

    (A) u is harmonic

    (B) u is continuous but not harmonic

    (C) u is only bounded

    (D) None of these

(Section-B)

(0.75 marks each)

Note: Attempt all questions.

1. Define critical point.
2. Define cross ratio.
3. State Cauchy-integral formula.
4. Define analytic function.
5. Define residue at infinity.
6. Define isolated singularity and give an example.
7. State Liouville's theorem.
8. State Taylor's theorem.
9. Find the residue of $\frac{1}{(z^2+a^2)^2} at\, z = ai$
10. State the argument principle.

(Section-C)

(1.25 marks each)

Note: Attempt all questions.

1. Evaluate the integral $\int_0^{1+i} (x - y + ix^2)dz$ along the straight line form z=0 to z=1+i

2. Consider the transformation $\omega = ze^{i\pi/4}$ and determine the region in the ω-plane corresponding to the triangular region bounded by the lines x=0,y=0 & x+y=1 in the z-plane.

3. Evaluate using Cauchy-Integral formula $\int_C \frac{\log z}{(z-1)^3} dz$

   where C is the circle |z-1|=½

4. Let f (z) be analytic within and on a circle C defined by $|z - z_0| = r$, If $|f(z)| \le M$ on C, then $|f^n(z_0)| \le n! \frac{M}{r^n}$

5. State and prove Cauchy-Residue theorem.

6. Let f(z) be an analytic function and let f '(z) be continuous at each point within and on a closed contour C, then $\int_C f(z)dz = 0$

7. Expand $f(z)=\frac{1}{z^2-3z+2}$ in a Laurent's series valid for the regin 1<|z|<2

8. Find the singularity of the function $\frac{e^{c/(z-a)}}{e^{(z/a)-1}}$ and indicate the character of each singularity.

9. Find the residue at the poles of function $\frac{z^4}{(c^2+z^2)^4}$

10. If a>e, use Rouche's theorm to prove that the equation $e^z=az^n$ has n-roots inside the circle |z|=1

P.T.O.

(Section-D)

(2 marks each)

Note: Attempt all questions.

1. Every Mobius transformation maps circles or straight lines into circles or straight lines

OR

Find the bilinear transformation which maps the points z_1=2, z_2=i, z_3=-2 into the points $\omega_1=1, \omega_2=i \& \omega_3 -1$

2. Let f (z) be an analytic function of z in a domain D of the z-plane and let f '(z) $\neq$ 0 inside D. Then the mapping $\omega = f(z)$ is conformal at all points of D.

OR

Use the method of contour integration to prove, that $\int_0^\infty \frac{\sin x}{x}dx=\frac{\pi}{2}$

3. Let f(z) be analytic within and on the boundary C of a simply connected region D and let *a* be any point within C, then derivatives of all orders are analytic and given by

$$f^n(a)=\frac{n!}{2\pi i}\int_C \frac{f(z)dz}{(z-a)^{n+1}}$$

OR

Show that sin $\left\{C\left(z+\frac{1}{z}\right)\right\}$ can be expanded in a series of the type $\sum_{n=0}^{\infty} a_n z^n + \sum_{n=0}^{\infty} bnz^{-n}$ in which the cefficients of both z^n & z^{-n} $are \frac{1}{2\pi}\int_0^{2\pi} \sin(2c\text{os}\,\theta)\cos n\theta d\theta$

4. State and prove Morera's theorem.

OR

Apply calculus of residue to prove that

$$\int_0^{2\pi} \frac{\cos 2\theta d\theta}{5+4\cos\theta} = \frac{\pi}{6}$$

5. State and prove Rouche's theorem.

OR

Use Rouche's theorem to show that the equation $z^5+15z+1=0$ has one root in the disc $|z|<3/2$ and four roots in the annulus $3/2<|z|<2$.